Estimation of von Bertalanffy Growth Parameters for the Walleye Population of Western Lake Erie

Richard W. Hatch

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Great Lakes Fishery Laboratory
U.S. Fish and Wildlife Service
Ann Arbor, Michigan 48105

ABSTRACT

Data on walleye length-at-age obtained in western Lake Erie by the resource management agencies of Michigan, Ohio, and Ontario, 1973-78, were combined in different ways to estimate the von Bertalanffy growth parameters K, L_∞ and t_0. Estimated parameters varied considerably when observations from spring and fall samples were treated separately; values for the combined data were: K = 0.22, L_∞ = 28.6 inches, and t_0 = -1.03. The Gulland maximum sustained yield coefficient, used by the Lake Erie Scientific Protocol Committee to estimate total allowable catch of walleyes from western Lake Erie, is considerably less variable, however, than the von Bertalanffy growth parameters; therefore, continued use of a value of 0.4 for this coefficient seems justified.

INTRODUCTION

Although increase in size of fish with age can be satisfactorily described in several ways (absolute increase, relative rate of increase, and instantaneous rate of increase), fitting growth data to a standardized mathematical model provides an additional way to compare growth among populations as well as between year classes, and simplifies the use of growth data in yield models. The von Bertalanffy growth curve is widely used in fishery studies because of its applicability to the dynamic pool model (Beverton and Holt 1957). Moreover, two of the parameters of the von Bertalanffy equation, K (the growth coefficient) and L_∞ (the asymptotic length of an average fish growing for an indefinitely long period of time), are used in computing Gulland's (1970) coefficient of maximum sustainable yield (MSY), the coefficient presently used in estimating harvestable surplus production of western Lake Erie walleyes.

In this report, I update Appendix Table 14 of the First Technical Report of the Great Lakes Fishery Commission Scientific Protocol Committee (SPC) on Interagency Management of the Walleye Resource of Western Lake Erie (Kutkuhn et al. 1976) by adding data from 1978, 1977 and 1976; estimate the von Bertalanffy parameters using length observations in different ways; compare estimates of growth parameters for individual year classes; and examine variation in the Gulland MSY coefficient as data are combined in different ways. In each analysis, save one, the von Bertalanffy parameters were estimated using the computer program BGC-2 (Abramson 1971).

DATA BASE

Data consisting of age-frequency distributions and mean length at age for walleyes caught in commercial trap nets and gillnets, experimental gillnets, and seines in spring and fall in western Lake Erie were submitted by the fisheries agencies of Michigan, Ohio, and Ontario, and by the U.S. Fish and Wildlife Service.

Spring samples contained 36 to 1,529 walleyes taken from 1973 through 1978; fall samples contained 52 to 892 walleyes taken from 1974 through 1978. Age-group 0 appears only in fall samples and mean length is biased upwards by gear selectivity. Age-group 1 is only sparsely represented in spring samples and mean length reported for this age group at that season is also probably biased upwards. For estimation of the parameter t_0, spring samples are considered to be taken before annual growth starts and fall samples are considered to be taken when 75% of annual growth has been completed since about three fourths of the annual temperature cycle in western Lake Erie is complete at the time fall samples are collected.

Observations of mean length at age submitted by each agency can be treated in a number of ways (i.e., pooling and weighting) for entry into a computer program to estimate the von Bertalanffy growth parameters. I compared four methods of combining the individual sample means; the treatments and their underlying assumptions are summarized as follows:

1) Replicate samples from the same population. Mean length-at-age estimates submitted by each agency for each year are considered replicate samples from the same population and entered into the program as individual observations. This treatment gives equal weight to each mean without regard to the number of observations used to calculate the mean. A second assumption in this treatment is that year-to-year differences in observed length at age result from random variation rather than actual changes in growth rate.

2) Annual weighted mean length. Mean length-at-age estimates submitted by each agency are weighted by sample size and combined to form a single estimate for each age in each year (Table 1). The annual weighted means are then entered into the program as replicate estimates of a population mean. This treatment also assumes that

year-to-year differences are random, but the data can be visually inspected for trends.

3) Pooled weighted mean length. Mean length-at-age estimates submitted by each agency are weighted as in 2), but are combined over years into a single mean length for each age.

4) Pooled unweighted mean length. Mean length-at-age estimates submitted by each agency in each year are considered as independent replicate samples from the same population, given equal weight, and averaged. This treatment tends to smooth out possible differences in growth of localized populations.

PARAMETER ESTIMATES

Results from Spring Observations

Estimates of the parameter K produced by treating the spring sample data according to the four methods above ranged from 0.230 to 0.280, L_∞ from 27.55 to 28.95 inches (700-735 mm) and t_0 from -0.61 to -0.94 years (Table 2). The mean estimates for each parameter (each mean weighted by the reciprocal of its variance) were: K = 0.242, L_∞ = 28.31 inches (719 mm), and t_0 = -0.80 years.

Results from Fall Observations

Samples obtained in fall were treated in the same four ways. Estimates of K ranged from 0.267 to 0.360, L from 24.90 to 26.85 inches (632-682 mm), and t_0 from -0.31 to -0.89 years (Table 2). The mean values for each estimate (weighted as before) were: K = 0.318, L_∞ = 25.68 inches (652 mm), and t_0 = -0.55 years. Age-0+ walleyes are proportionately more common in fall samples than age-1 walleyes in spring samples, and age-5+ walleyes are less abundant in fall samples than age-6 walleyes in spring samples. When the growth parameters are estimated from fall samples, the seasonal age composition differences depress the left limb of the curve and cause a more rapid approach to the asymptote resulting in a higher value of K, a t_0 closer to the origin, and a smaller value for L_∞.

Spring and Fall Samples Combined

Spring and fall estimates of combined weighted mean length (treatment 3) were used together to form a single growth curve (Fig. 1) comparable to that included in Addendum 1 to the SPC Report. The combined sample contained over 13,000 walleyes ranging in age from 0+ to 8 years, with 96% of the observations from fish age 1+ to 5 years. (Again, walleyes in fall samples were considered to have completed 75% of that year's growth.) The estimated parameters of this curve ($K = 0.22$, $L_\infty = 28.6$ inches [726 mm], $t_0 = -1.03$ years) probably best represent average growth during the period 1970 through 1976. [Computer program BGC-2, used to estimate parameters for spring and fall samples, requires equally spaced age groups (i.e., 1 year between samples) but allows use of unequal sample sizes for separate ages. When spring and fall samples are combined, age groups are not equally spaced (i.e., fall samples are considered 3/4-year older than spring samples of the same age group; the same cohort sampled in spring is considered 1/4-year older than it was the preceding fall). Thus, program BGC-2 could not be used for the combined data. Instead, these unequally spaced means were processed by the first section of program WVONB (Pienaar and Thomson 1973), which does not yield standard errors of the estimates.]

Growth Curves for Individual Year-Classes

Comparison of growth curves fitted to individual year-classes provides a method for detecting changes in growth with time. Because of the short time span for which data were available, the growth curves of only three cohorts could be fitted: 1970 year-class, ages 3-8 ($n = 1608$); 1971 year-class, ages 2-7 ($n = 525$); and 1972 year-class, ages 2-6 ($n = 722$). All three K values estimated from individual year classes (Table 3) are higher than those obtained from mean growth curves (Table

2) but confidence intervals for the former estimates are quite wide. This high variability obscures any real differences between year classes.

Variation in the MSY Coefficient

The maximum sustained yield coefficient X proposed by Gulland (1970), i.e, "maximum sustained yield expressed as a percentage of M times unfished biomass" which may safely be harvested, is determined by the ratio of size at first capture to L_∞ and the ratio of the natural mortality coefficient M to the growth coefficient K. Values for X calculated from the several treatments of spring and fall sample mean lengths, and from the combined spring and fall data, ranged from 0.42 to 0.54 when size at entry is 16.5 inches (419 mm) and the natural mortality coefficient is 0.218 as determined in the SPC report. The small range of variation is to be expected, given the reciprocal relationship between K and L_∞. The median value for X was 0.44.

Length at age 2 calculated from the von Bertalanffy growth curves ranged from 10.5 to 14.8 inches (267-376 mm), depending on the separate or combined data sets used (median value = 14.1 inches [358 mm]). If these values are used for length at entry into the catch, values for X range from 0.33 to 0.43 with a median of 0.38. Estimates of length at age 2+ (fall) ranged from 13.7 to 17.2 inches (348-437 mm) (median = 16.4 inches [417 mm]) and corresponding X values from 0.39 to 0.48 with a median of 0.43.

CONCLUSIONS AND RECOMMENDATIONS

1) Estimated parameters of von Bertalanffy growth curves vary considerably when sample means are combined in various ways. Median values for the three estimated parameters are $K = 0.27$, $L_\infty = 27.6$ inches (701 mm), $t_0 = 0.77$ years. Values for all data combined (weighted means) are: $K = 0.22$, $L_\infty = 28.6$ inches (726 mm), $t_0 = -1.03$ years.

2) Development of methods to obtain unbiased spring samples of age-1 walleyes would considerably improve the estimation of growth parameters.

3) The Gulland maximum sustainable yield coefficient X is considerably less variable than the von Bertalanffy growth parameters because of the reciprocal relationship between K and L_{∞}. Continued use of X = 0.4 in estimating total allowable catch of walleyes from western Lake Erie seems justified.

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Table 1. Mean length (inches) of western Lake Erie walleyes sampled in spring (integer ages) and fall (75% of a growth year completed) 1973-1977. Number of walleyes contributing to each sample in parentheses.

Age (years)	1973	1974	1975	1976	1977	1978	Weighted mean
0.75	-	8.4 (7)	8.2 (14)	9.2 (2)	9.5 (61)	9.2 (10)	9.2 (94)
1.00	-	10.7 (15)	-	9.0 (2)	9.0 (6)	10.0 (55)	10.0 (78)
1.75	-	14.4 (53)	13.6 (299)	13.6 (510)	14.4 (83)	14.0 (2457)	13.9 (3402)
2.00	14.6 (206)	15.1 (268)	14.9 (65)	14.2 (410)	14.0 (911)	15.0 (220)	14.4 (2080)
2.75	-	17.5 (17)	17.6 (41)	17.1 (243)	16.7 (539)	17.2 (105)	16.9 (945)
3.00	17.6 (1172)	18.1 (155)	18.2 (152)	17.8 (54)	17.2 (726)	17.0 (1787)	17.3 (4046)
3.75	-	19.5 (14)	20.4 (13)	19.7 (29)	18.7 (152)	18.7 (484)	18.8 (692)
4.00	20.2 (148)	20.5 (275)	20.1 (42)	19.8 (84)	19.2 (284)	18.9 (763)	19.4 (1596)
4.75	-	21.0 (3)	22.4 (4)	22.1 (7)	20.7 (17)	20.0 (162)	20.2 (193)
5.00	21.8 (10)	22.1 (71)	22.1 (54)	21.0 (31)	21.4 (132)	20.5 (94)	21.4 (392)
5.75	-	23.5 (2)	22.6 (1)	22.1 (6)	22.4 (5)	21.5 (19)	21.9 (33)
6.00	23.9 (2)	23.8 (14)	24.5 (6)	22.3 (37)	22.5 (72)	22.5 (86)	22.6 (217)
7.00	23.5 (2)	25.4 (5)	25.8 (1)	23.9 (7)	24.5 (54)	23.2 (19)	24.2 (88)
8.00	-	28.3 (1)	-	25.6 (1)	25.5 (8)	25.1 (16)	25.4 (26)

Table 2. Estimated parameters of the von Bertalanffy growth curve for walleye samples from western Lake Erie. Individual sample means combined in various ways (see text for explanation). Standard error of the estimate in parentheses.

Data treatment and season	K	L_∞ inches	t_0 years
1S) Individual spring means as replicate samples	0.230 (±0.028)	28.75 (±1.13)	-0.92 (±0.21)
2S) Annual weighted spring means as replicate samples	0.280 (±0.028)	27.55 (±0.81)	-0.61 (±0.16)
3S) Spring combined weighted mean length	0.227 (±0.022)	28.95 (±0.90)	-0.94 (±0.16)
4S) Spring combined unweighted mean length	0.246 (±0.030)	28.33 (±1.06)	-0.77 (±0.19)
1F) Individual fall means as replicate samples	0.302 (±0.027)	26.67 (±0.74)	-0.65 (±0.12)
2F) Annual weighted fall means as replicate samples	0.360 (±0.028)	24.90 (±0.53)	-0.49 (±0.10)
3F) Fall combined weighted mean length	0.267 (±0.051)	26.66 (±1.64)	-0.89 (±0.27)
4F) Fall combined unweighted mean length	0.292 (±0.050)	26.85 (±1.42)	-0.31 (±0.22)
Spring and fall weighted mean length used concurrently	0.223 (*)	28.61 (*)	-1.03 (*)

*Estimates of standard errors were not calculated for seasons combined because of unequally spaced time intervals.

Table 3. Estimated parameters of the Von Bertalanffy growth curve for three year classes of western Lake Erie walleyes. Standard error parentheses.

Year Class	K	L_∞ inches	t_0 years
1970, ages 3-8	0.28 (±0.16)	27.3 (±2.8)	-0.76 (±1.6)
1971, ages 2-7	0.39 (±0.07)	24.4 (±0.8)	-0.34 (±0.35)
1972, ages 2-6	0.34 (±0.07)	25.0 (±1.1)	-0.76 (±0.40)

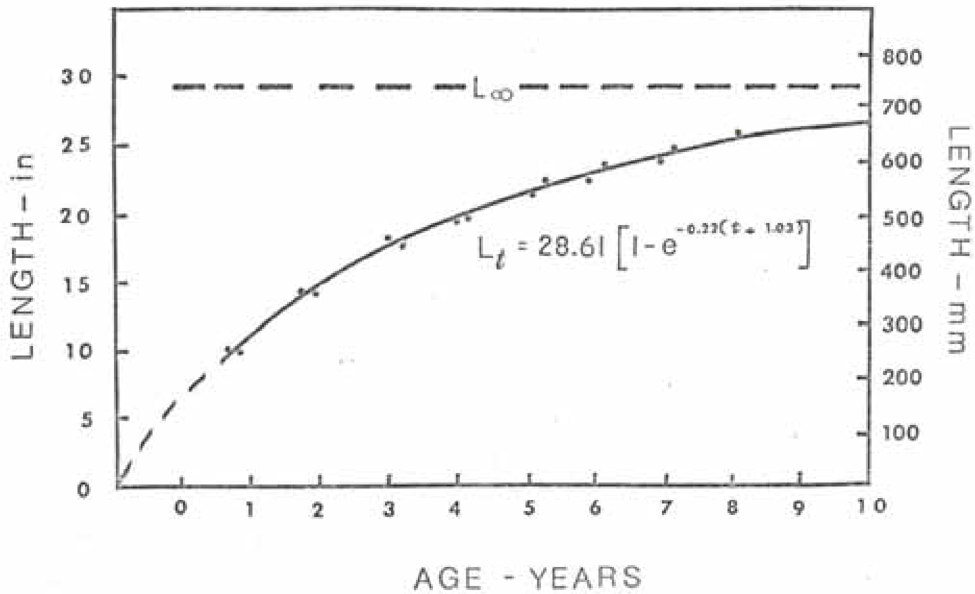

Figure 1. Growth curve for western Lake Erie walleyes derived from samples taken by trap nets, gillnets, and seines, 1973-77. Points indicate approximate locations of spring and fall sample means.